

Application of some common water saturation formulations in 3D reservoir models

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1 Introduction

This memo treats some common formulations for saturation versus height, which are commonly applied in reservoir modelling. For details on basic principles in capillary pressure etc., confer text books on the topic or e.g. [AAPG pages](#).

Equations are implemented in Equinor's open source python library [fmu-tools](#).

1.1 First, about the normalized capillary pressure

At this point we can introduce the term *normalized capillary pressure*, which is

$$P_{CN} = \frac{P_C}{\sigma \cos \theta} = \frac{(\rho_w - \rho_{hc})gh}{\sigma \cos \theta} = \frac{\Delta\rho \cdot g \cdot h}{\gamma_{hc}} \quad (1)$$

Here γ_{hc} is named the adhesion tension. If we instead of density ρ and gravity constant g use fluid gradients $\varrho = \rho \cdot g$, we get:

$$P_{CN} = \frac{\Delta\varrho \cdot h}{\gamma}, \quad h = FFL - TVD \quad (2)$$

Here FFL is the free fluid level for a given phase, and TVD is the actual depth. This is normally only evaluated for $h \geq 0$.

In an oil-water-gas system this becomes for oil:

$$P_{CN,o} = \frac{\Delta\varrho_{wo} \cdot h}{\gamma_{wo}}, \quad h = FWL - TVD \quad (3)$$

And for gas, we add the contribution from the oil leg:

$$P_{CN,g} = \frac{\Delta\varrho_{wo} \cdot H_o}{\gamma_{wo}} + \frac{\Delta\varrho_{wg} \cdot h}{\gamma_{wg}} \quad (4)$$

where H_o is a constant, while h is the variable to compute across, i.e.

$$H_o = FWL - FOL$$
$$h = FOL - TVD$$

Hence we can say that:

$$P_{CN,g} = M + \frac{\Delta\varrho_{wg} \cdot h}{\gamma_{wg}} \quad \text{where } M \text{ is constant for a given 3D cell} \quad (5)$$

1.2 Next, about normalization

In some cases, use of minimum and maximum (more rarely) saturations are relevant. In particular, a minimum value $S_{swir,a}$ is useful. This $S_{swir,a}$ must not be confused with S_{swirr} (minimum saturation, SWL) as applied in reservoir simulation studies. Hence:

$$S_{w,final} = S_{wir,a} + (S_{w,max} - S_{wir,a}) * S_w \quad (6)$$

Hence, it is the normalized S_w that is the culprit to compute, and a more correct term for this would be S_{wN} , but for simplicity we call this S_w in the following.

2 The Leverett J formulation

The Leverett J is the most known saturation height function, and is commonly formulated as

$$S_w = A \cdot J^B \quad (7)$$

It can also be formulated in an inverse manner, as

$$S_w = \left(\frac{J}{\hat{A}} \right)^{\frac{1}{\hat{B}}}$$

In this case, the conversions are $A = (1/\hat{A})^{1/\hat{B}}$ and $B = 1/\hat{B}$.

The J here, has two variants, a simplified ("log-based"):

$$J = h \sqrt{\frac{k}{\phi}} \quad (8)$$

Here h is the *height term* (which is the important variable to integrate across vertically, see later), while k is permeability and ϕ is porosity. Note that the permeability term here is not necessarily the "dynamic" flow permeability, but a "local" permeability *dedicated* to water saturation calculations. In some cases, this permeability is derived fully from porosity (a perm-poro relationship).

In contrast a full J (also termed "core-based") is

$$J = \frac{\bar{C}(\rho_w - \rho_{hc})gh}{\sigma \cos \theta} \sqrt{\frac{k}{\phi}} = \frac{\bar{C}(\rho_w - \rho_{hc})h}{\gamma} \sqrt{\frac{k}{\phi}} \quad (9)$$

Here \bar{C} is a conversion factor to make it unitless.

Now remember the definition of P_{CN} , and apply that in the formula:

$$J = \bar{C}P_{CN} \sqrt{\frac{k}{\phi}} \quad (10)$$

Which in turns makes, for oil leg:

$$S_{w,o} = A \left(\frac{\bar{C} \Delta \rho_{wo} \cdot h}{\gamma_{wo}} \sqrt{\frac{k}{\phi}} \right)^B \quad (11)$$

and for gas leg:

$$S_{w,g} = A \left(\left(\frac{\bar{C} \Delta \rho_{wo} \cdot H_o}{\gamma_{wo}} + \frac{C \Delta \rho_{wg} \cdot h}{\gamma_{wg}} \right) \sqrt{\frac{k}{\phi}} \right)^B \Rightarrow$$

$$S_{w,g} = A \left(\bar{C} \cdot (M + P_{CN,g}) \sqrt{\frac{k}{\phi}} \right)^B \quad (12)$$

A generic equation is then:

$$S_w = a(m + x \cdot h)^b \quad (13)$$

for full Leverett J using normalized capillary pressures:

$a = A =$ constant regression parameter

$b = B =$ constant regression parameter

$$m = 0 \quad (\text{oil}) \quad \text{or} \quad \frac{\bar{C} \Delta \rho_{wo} \cdot H_o}{\gamma_{wo}} \sqrt{\frac{k}{\phi}} \quad (\text{gas, if oil below})$$

$$x = \bar{C} \cdot \frac{\Delta \rho_{w,hc}}{\gamma_{w,hc}} \sqrt{\frac{k}{\phi}}$$

where for simplified Leverett J:

$a = A =$ constant regression parameter

$b = B =$ constant regression parameter

$$m = 0$$

$$x = \sqrt{\frac{k}{\phi}}$$

3 The BVW formulation

The BVW (Bulk Volume Water) formulation can be formulated as

$$S_w = A \cdot P^B \cdot \phi^C \quad (14)$$

Here A, B, C are constants per 3D cell, and are e.g. derived as:

$$A = a1 \cdot \phi + a2$$

$$B = b1 \cdot \phi + b2$$

$$C = c1 \cdot \phi + c2$$

In many cases, P term is P_{CN} (the normalized capillary pressure), i.e.

$$P_{CN} = \frac{(\rho_w - \rho_{hc})h}{\gamma_{hcw}} = \frac{\Delta\rho \cdot h}{\gamma} \quad (15)$$

Hence we can re-arrange into a generic expression:

$$S_w = a \cdot (xh)^b \quad (16)$$

As earlier stated for the oil-water-gas systems, the P_{CN} may be expressed as

$$P_{CN} = \frac{(\rho_w - \rho_o)H_o}{\gamma_{ow}} + \frac{(\rho_w - \rho_g)h}{\gamma_{gw}} \quad (17)$$

As above, H_o is a constant, hence the whole system can be written as

$$S_w = a \cdot (m + xh)^b \quad (18)$$

which is exactly of the same form as 13! But in contrast to a J function, we have here:

$$a = A \cdot \phi^C$$

$$b = B$$

$$m = 0 \quad (\text{oil}) \quad \text{or} \quad \frac{\Delta\rho_{wo} \cdot H_o}{\gamma_{wo}} \quad (\text{oil term in gas})$$

$$x = \frac{\Delta\rho_{w,hc}}{\gamma_{w,hc}} \quad (\text{"PCN without height"})$$

4 The Brooks-Corey formulation

There seems to be several fomulation for the Brooks-Corey equation, but one example is:

$$S_{w,final} = S_{wi} + (1 - S_{wi}) \cdot \left(\frac{P_{CN,e}}{P_{CN}} \right)^{\frac{1}{N}} \quad (19)$$

$$S_w = \left(\frac{P_{CN,e}}{P_{CN}} \right)^{\frac{1}{N}} = (P_{CN,e})^{\frac{1}{N}} \cdot (P_{CN})^{\frac{-1}{N}} \quad (20)$$

$S_{w,final}$ = Resulting saturation
 S_w = Normalized saturation
 S_{wi} = Irreducible water saturation, as constant or a function of e.g. porosity
 $P_{CN,e}$ = Entry value of normalized capillary pressure, as constant or a function
 P_{CN} = Normalized capillary pressure
 N = Corey exponent, as constant or a function of .e.g. porosity

Parameters can be for example:

$$\begin{aligned}
 S_{wi} &= a1 \cdot \phi + a2 \\
 P_{CN,e} &= b1 \cdot \phi^{b2} \\
 N &= c1 \cdot \phi + c2
 \end{aligned}$$

By reformulation, also this one can written as:

$$S_w = a(m + xh)^b \tag{21}$$

Here

$$\begin{aligned}
 h &= \text{The height above the free fluid level} \\
 a &= (P_{CN,e})^{1/N} \\
 m = 0 \quad \text{oil, or for gas with oil below:} \quad m &= \frac{\Delta \rho_{wo} \cdot H_o}{\gamma_{wo}} \\
 x &= \frac{(\rho_w - \rho_{hc})}{\gamma_{hc,w}} \\
 b &= -\frac{1}{N}
 \end{aligned}$$

5 Integration of height function

In order to work effiecently in 3D models, one should allow having thick cells in some or all zones. In particular redermination projects work on zone averages, which would give rather thick cells in practice. However, when computing water saturation as a height-above-contact function, thick cells may lead to imprecise results when just using the cell center (i.e. a direct method). There are then two options left:

1. Refine the model vertically in order to mimic "numerical integration"
2. Implement analytical integration of the saturation functions

To refine the model vertically just for the purpose of water saturation modelling is not optimal, as it would lead to significantly longer computing time, and also slow the graphical performace. Hence, using analytical integration is the better choice. To integrate the generic function is not difficult:

$$\begin{aligned}
S_w &= a \cdot (m + xh)^b \Rightarrow \int S_w dh \Rightarrow \\
&\int_0^{H_T} 1 dh + \int_{H_T}^{H_2} a(m + xh)^b dh \Rightarrow \\
&\frac{1}{\Delta H} \left[H_T + \frac{a}{x(b+1)} \left((m + xH_2)^{b+1} - (m + xH_{1*})^{b+1} \right) \right]
\end{aligned}$$

5.1 Determinating threshold height H_T

The threshold pressure height can be determined with

$$S_w = a \cdot (m + xh)^b = 1$$

$$H_T = \left(\left(\frac{1}{a} \right)^{\frac{1}{b}} - m \right) \frac{1}{x} \quad (22)$$

Note that this height is independent of $S_{wir,a}$ and $S_{w,max}$ scaling. Note also that H_2 and H_{1*} can be computed in different ways, as illustrated in next section. The star * in H_{1*} is applied since it may be set to H_T or base of cell, dependent on position of the contact in the cell.

6 Geometrical considerations

When either finding the "cell center" or the cell base/top to integrate across, there are several choices as shown in Figure 1. The geometries will further depend if just cell mid is pllaied (direct method), or an integration is applied.

The various combinations may give different results (see examples in Figure 2 in , and in general, using integration and "cell corners above FFL" is the best approach for volumetric accuracy.

7 Final note

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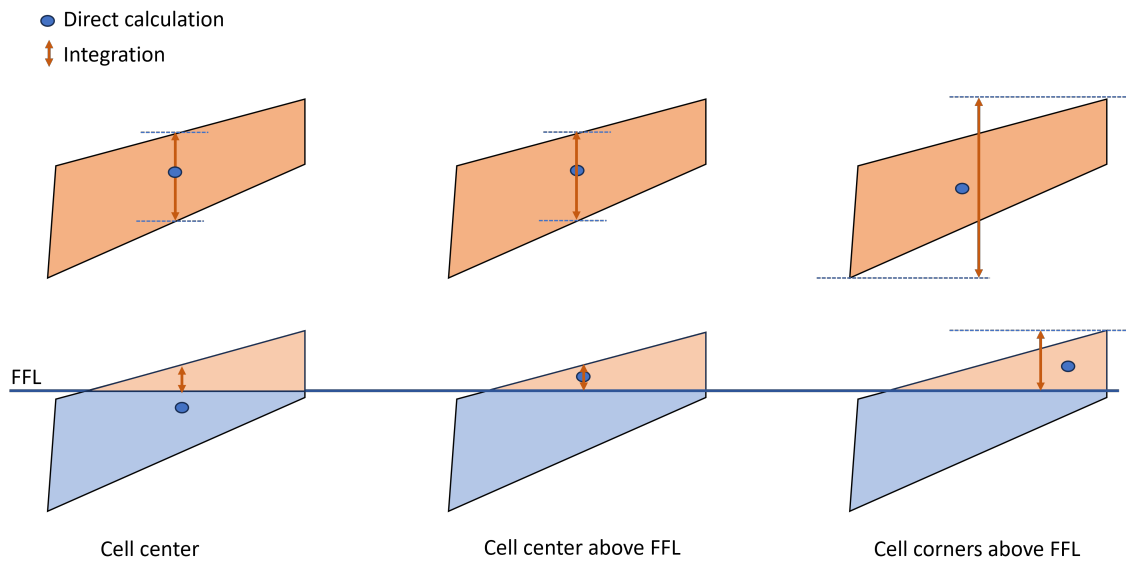
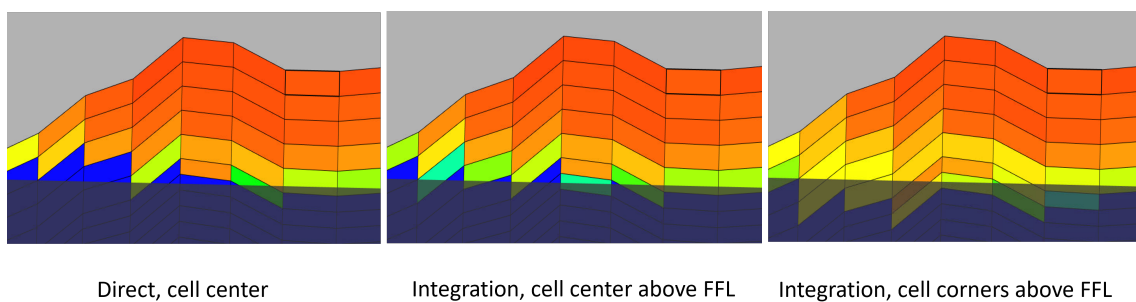


Figure 1: Illustration on how geometrical approaches may vary, in combination with a direct approach, or using integration.



Note that some cells appear "non-logical", but remember cells are in 3D where only the front is shown here

Figure 2: Examples from different settings.